EE 120 Signals & Systems (16B Reimagined + Fourier Transform)

Circuits: Phasors → Fourier Transform

Sampling: CT/DT

Controls: Input → $\boxed{\text{System}}$ → output

EE 127: Optimization

Cost: $f(x)$

Constraints: $g(x) < \alpha$

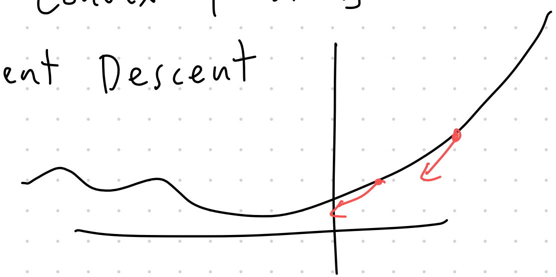
$h(x) = \beta$

$\vdots$

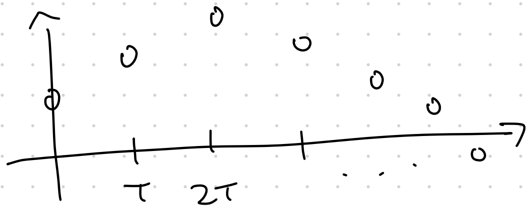
SVD, Linear Progamming,

Convex Problems

Gradient Descent



Sampling



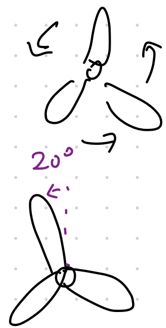
- What is the sampling rate? T
- Sampling freq: $w = \frac{2\pi}{T}$
- At what frequency do we need to sample at for a perfect reconstruction?

$w_s > 2w_{max}$

HW

Real Time

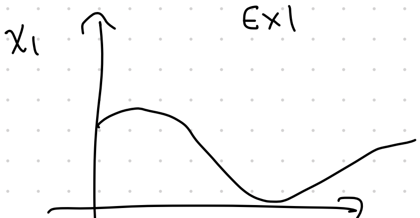
Video



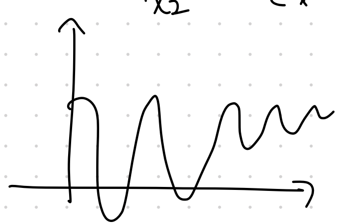
In one frame the spinner made $\frac{17}{18}$th of a revolution

Sampled & Reconstructed Signals

Ex1

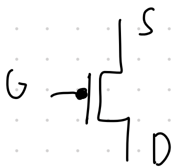


Ex 2

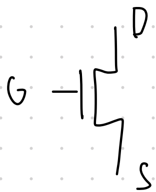


- Which example has the quicker sampling rate?

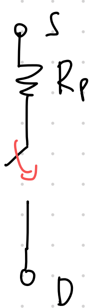
MOSFETs:



PMOS



NMOS

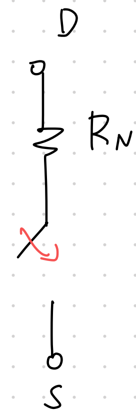


PMOS switch turns on when

$V_{SG} = V_S - V_G > |V_{tp}|$

↑ PMOS Threshold

Absolute Value because V_{tp} is negative



NMOS switch turns on when $V_{GS} = V_G - V_S > |V_{tn}|$

↗ NMOS Threshold

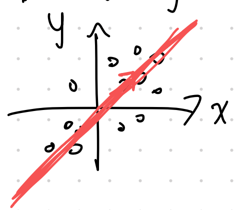
$\sigma_i = \sqrt{\lambda_i}$ where λ_i is an eigenvalue of A^TA

PCA / SVD

SVD: $A = U \Sigma V^T$

vectors in the V matrix are the eigenvectors of A^TA

PCA: Way to perform dimensionality reduction on data



$$A = \begin{array}{c c} & \begin{array}{cc} x & y \end{array} \\ n & \begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{bmatrix} \\ & \begin{array}{cc} & 2 \end{array} \end{array}$$

Covariance Matrix

$$C = \frac{1}{n} \tilde{A}^T \tilde{A}$$

$\tilde{A}$ is the demeaned matrix $\tilde{A} = A - \frac{1}{n} 1 1^T$

$$C = \frac{1}{n} \; {}^{2}\begin{bmatrix} Var(x) & Cov(x,y) \\ Cov(x,y) & Var(y) \end{bmatrix}_{2}$$

The principal components that maximize variance are the eigenvectors of C, variance along pc_i is λ_i

SVD + PCA:

Given a matrix A, 1. we can construct $B = \frac{1}{\sqrt{n}} \tilde{A}$

2. Take the SVD of $B = U \Sigma V^T$ ← eigenvectors of $B^TB = \left(\frac{1}{\sqrt{n}} \tilde{A}^T\right)\left(\frac{1}{\sqrt{n}} \tilde{A}\right)$

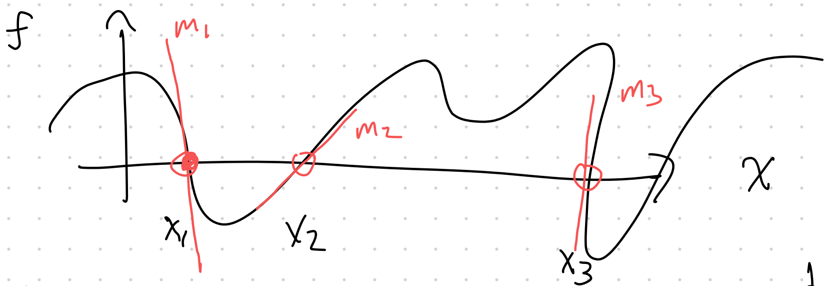
$$C = \frac{1}{n} \tilde{A}^T \tilde{A}$$

3. The vectors in V are the P.C. s
4. Singular values σ_i represent the std deviation along v_i

Equilibrium / Stability

$\frac{d}{dt}\vec{x} = f(\vec{x}, \vec{u})$, the system is at equilibr. when $\frac{d\vec{x}}{dt} = \vec{0}$

$f(\vec{x}) = \vec{0}$



x_1, x_2, x_3 are eq. pts.

$$\frac{d}{dt}x = f(x)$$

$$\frac{d}{dt}(x - x_1) = m_1 \cdot (x - x_1)$$

eigenvalue

$$\frac{dx}{dt} = \lambda x$$

Linearization says

$$\frac{d}{dt}x_\ell = J_x \cdot x_\ell + J_u \cdot u_\ell$$

Open Loop Control $\quad \vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t) \quad (1)$

System

Given inputs $\vec{u}(0), \vec{u}(1), \ldots, \vec{u}(k)$,

how can we reach a target $\vec{x}(k+1) = \vec{x}_{target}$

if we want to reach the state $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ in 4 time-steps

what inputs $\vec{u}(0), \ldots, \vec{u}(3)$ should I give?

$$\vec{x}(4) = A^4\vec{x}(0) + A^3B\,\vec{u}(0) + \ldots + B\vec{u}(3)$$

$$\underbrace{\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} - A^4\vec{x}(0)}_{\text{vector}} = \begin{bmatrix} A^3B & A^2B & AB & B \end{bmatrix} \begin{bmatrix} \vec{u}(0) \\ \vec{u}(1) \\ \vec{u}(2) \\ \vec{u}(3) \end{bmatrix}$$

solve for $\vec{u}(k)$

Closed Loop Control, use a feedback policy $\vec{u}(t) = K\vec{x}(t)$

$$\vec{x}(t+1) = A\vec{x}(t) + BK\vec{x}(t) = (A+BK)\vec{x}(t)$$

$\vec{u}(t) = \vec{x}_{target} + K\vec{x}(t)$

If (1) is controllable, then we can assign the eigvals of $A+BK$ arbitrarily

If stable $\vec{x}(t) \to \vec{0}$