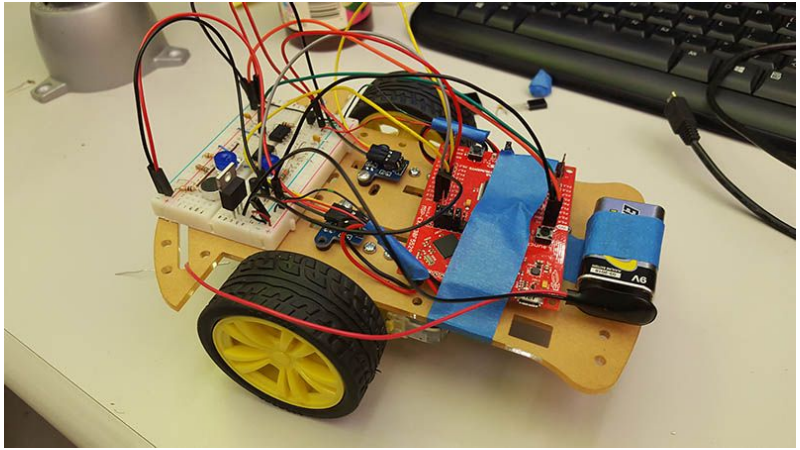
Logo credits go to Moses Won

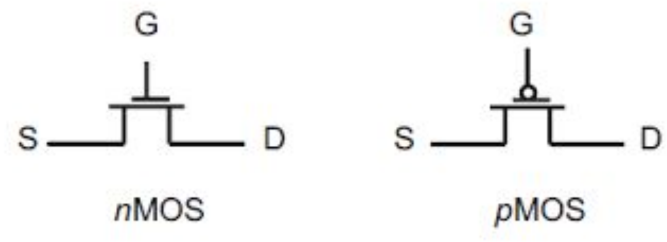
Discussion 14B

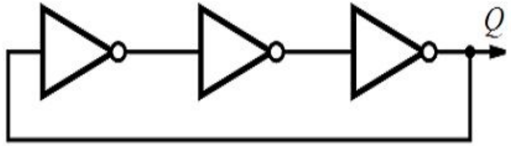
Course Recap



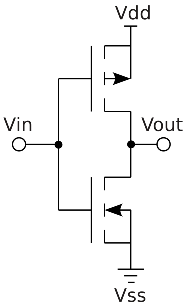
Circuits

PMOS / NMOS Transistors (Digital Circuits):





Underlying Circuit Model was an RC Circuit.



Linear Algebra & Differential Equations

Differential equations were the language to model systems.

$$\frac{dx}{dt} + ax = 0$$

$x(0) = x_0$ ↑ Initial Condition

$\frac{dx}{dt} = -ax$

$x(t) = x_0 e^{-at}$

We used Linear Algebra to solve any **linear** differential equations.

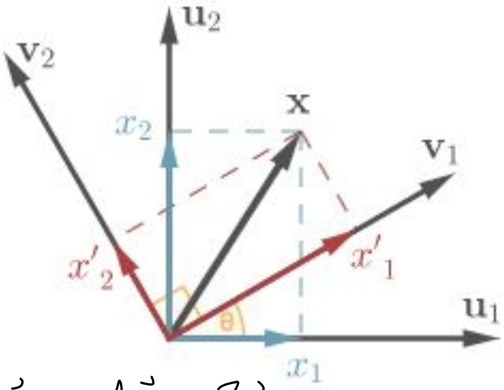
$$\frac{d}{dt}x(t) = \lambda x(t) + u(t)$$

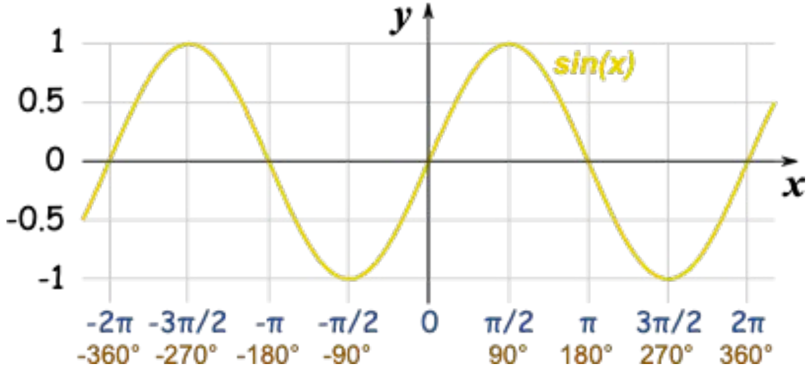
$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t) + \vec{b}$$

$\vec{x}(t) = \alpha_1 e^{\lambda_1 t} \vec{v}_1 + \dots + \alpha_n e^{\lambda_n t} \vec{v}_n$

Changing Coordinates / Domains

If a problem is difficult, we can always change the problem: to make it easier





Time

↓

Frequency/

Phasors

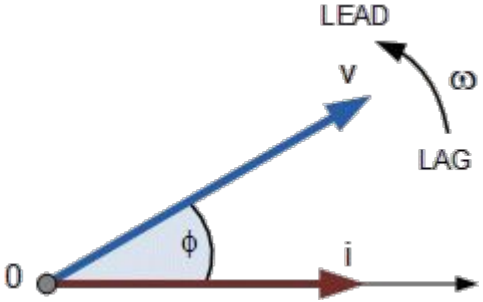
$$\frac{d}{dt}\vec{x} = A\vec{x} + B\vec{u}$$

Change of Basis

$$\frac{d}{dt}\vec{z} = \Lambda\vec{z} + \tilde{B}\vec{u}$$

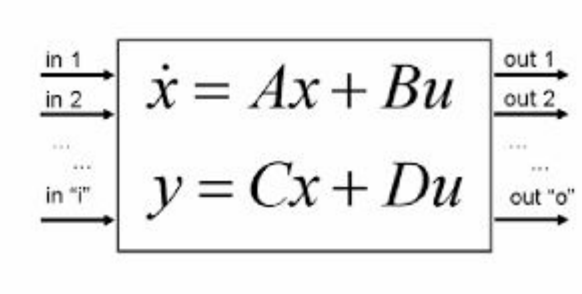
↑

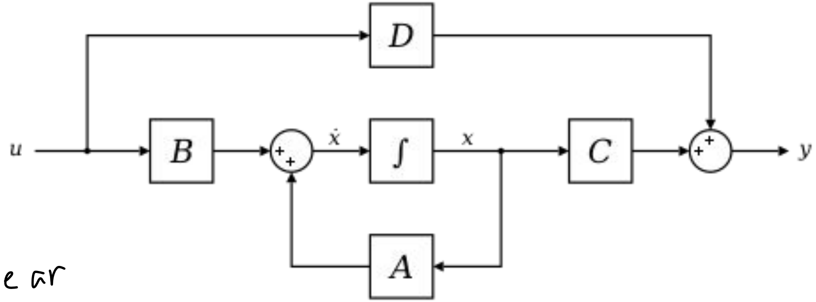
diagonal



Controls

We first had to understand how systems behave through the State-Space Model





Linear

$$\frac{d\vec{x}}{dt} = A\vec{x} + B\vec{u}$$

Non-linear

$$\frac{d\vec{x}}{dt} = f(\vec{x}, \vec{u})$$

System Properties:

1. Linearity
2. Stability
3. Controllability

Every Bounded Inputs have Bounded Outputs?

Eigenvals of A: $\mathrm{Re}(\lambda) < 0$ (CT) or $|\lambda| < 1$ (DT)

Can we reach any state?

Controls

Once we understood how systems could behave, we were able to control them.

If the system is controllable, we can reach anywhere in our state-space.

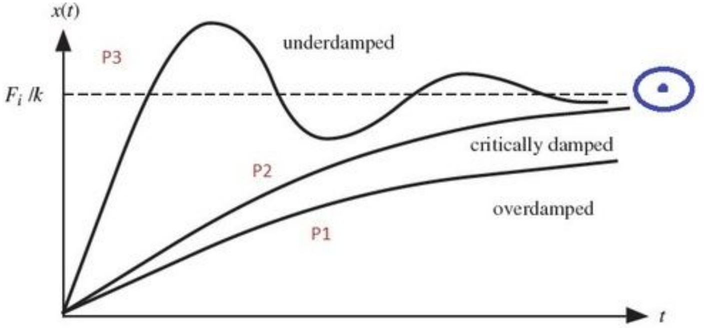
- More importantly, we can also pick all of the eigenvalues of our system using **feedback control.**
- This lets us control the **shape** of our response.

$u_0, u_1, \dots, u_n$

$\rightarrow \boxed{\text{System}} \rightarrow$

Outputs

$\vec{x}(0), \dots, \vec{x}(n)$



SVD & Optimization

The SVD let us decompose a matrix into its most important parts:

and least

$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \ldots + \sigma_k \vec{u}_k \vec{v}_k^T = \sum_{i=1}^{k} \sigma_i \vec{u}_i \vec{v}_i^T$$

weights

How can we minimize / maximize some cost subject to constraints?

$$\min_{\vec{x} \in \mathbb{R}^n} \|\vec{x}\|^2 \quad \text{subject to } A\vec{x} = \vec{y} \qquad \max_{\{\vec{x} : \|\vec{x}\| = 1\}} \|A\vec{x}\| \rightarrow \|U \Sigma V^T \vec{x}\|$$

cost

cost

$\min \|\vec{x}\|$ subject to $C\vec{x} = \vec{y}$

$\vec{x} = C^{+}\vec{y}$

$$\vec{x}(t+1) = A\vec{x}(t) + B\vec{u}(t)$$

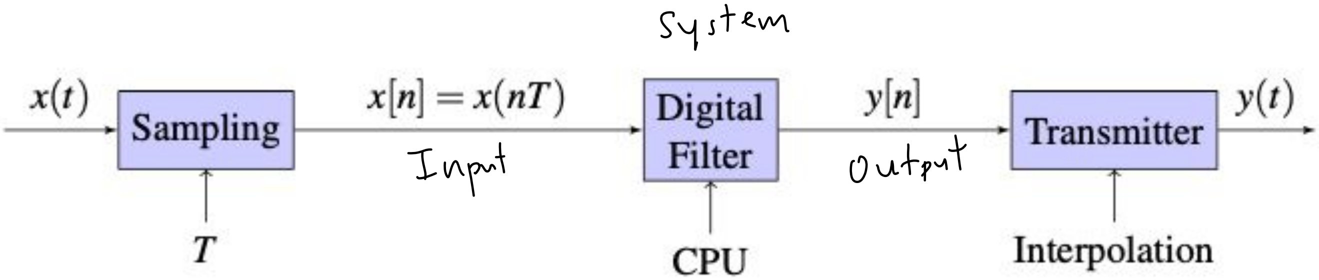
$$\vec{x}(k) = A^k \vec{x}(0) + A^{k-1} B \vec{u}(0) + \ldots + B u(k-1)$$

$$\underbrace{\vec{x}(k) - A^k \vec{x}(0)}_{\vec{y}} = \underbrace{\begin{bmatrix} A^{k-1}B & \cdots & A^{k-2}B & \cdots & AB & B \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} u(0) \\ \vdots \\ u(k-1) \end{bmatrix}}_{\vec{x}}$$

Signals and Systems

Most real world systems can be modeled through an input/output box model:

Interfacing between CT and DT:



What's Next?

If you liked the circuits portion:

- Transistors: EE 105 (Microelectronic Circuits), EE 140 (Analog Design)
- Device Physics: EE 130s (Device Physics)
- Digital Circuit Design: CS 61C, EECS 151 (Digital Design & IC)
- Power Electronics: EE 137A/B (Power Systems)

CS 152

What's Next?

Controls: EE 120 (Signals & Systems), EE 128 (Control Systems)

Robotics: EECS 106A/B, EE 128, EE 192 (Mechatronics Lab)

Optimization / Linear Algebra: EE 127 (Optimization)

Machine Learning: EE 126 (Probability) / Prob 140, Data 100, EE 127

Digital Signal Processing: EE 120 (Signals & Systems), EE 123 (DSP), EE 126

Software: CS 170 (Algorithms), CS 186 (Databases), CS 162 (OS & Systems), CS 161 (Security)

Math: Math 110 (Linear Algebra), Math 104 (Real Analysis), Math 123 (Ordinary Differential Equations), Math 185 (Complex Analysis)