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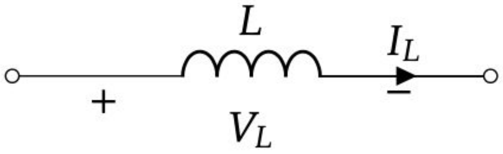
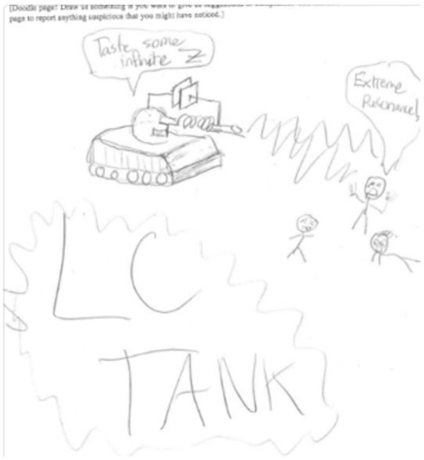
Discussion 4B

Inductors & LC Tanks





Recap from Last Week

We saw how to solve vector equations of the form: $\mathbf{x}'(\mathbf{t}) = \mathbf{A}\ \mathbf{x}(\mathbf{t})$.

- We used diagonalization to create a new equation $\mathbf{z}'(\mathbf{t}) = \mathbf{\Lambda}\ \mathbf{z}(\mathbf{t})$.
- Each $z_i(t)$ could be solved as a first-order diff-eq.
- We converted our solution back to $\mathbf{x}(\mathbf{t})$ using $\mathbf{x}(\mathbf{t}) = \mathbf{V}^{-1}\ \mathbf{z}(\mathbf{t})$
- This showed that the solution was a linear combination of "eigenfunctions" $e^{\lambda_i t}$

Then we took a look at Complex Numbers

- Complex numbers are necessary to model rotations and oscillations in the real world.

Overview for Today

Today we look at an **inductor** and how it behaves with a capacitor.

1. Inductors behave in the opposite way as capacitors.
    a. $V_L = L\, dI_L/dt$ while $I_c = C\, dV_c/ dt$
2. An inductor and capacitor creates an LC Tank which has **complex eigenvalues.**
3. We will look at the energy across the inductor and capacitor and see oscillations.

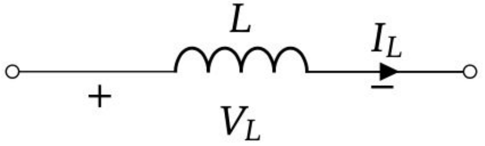
Inductors

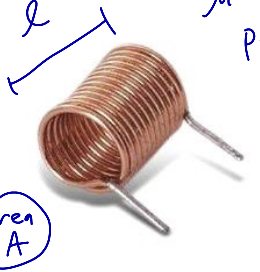
$L = \frac{\mu N^2 A}{l}$

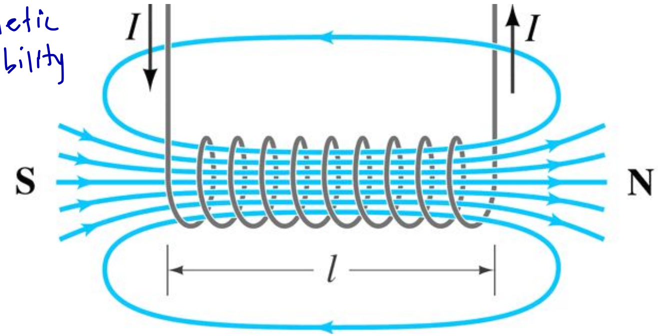
N: # of turns
A: cross sectional area

An inductor is an electrical device that stores magnetic **flux** when a current flows through it.

μ: magnetic permeability







$C = \frac{\varepsilon_0 A}{d}$

ε_0: permittivity constant

Flux is the amount of flow through a surface. The blue lines in the third diagram represent the magnetic flux through an inductor. The greek letter Φ to denote flux.

Similar to how Q = CV for a capacitor, Φ = LI for an inductor.

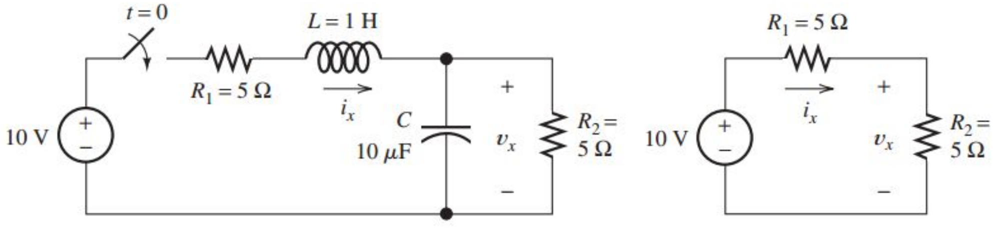
I-V Relations for an Inductor

$V_L = \frac{dI_L}{dt} = 0$

What is the I-V Relation for an inductor?

- We stated in the last slide that $\Phi = LI$.
- Faraday's Law from Physics tells us that $d\Phi/dt = V$.
- Therefore, we conclude that $V = d\Phi/dt = L\, dI/dt$.

Recall that capacitors behave like open circuits at steady state. Inductors however, behave like **shorts** since $dI/dt = 0$ so $V_L = 0$.

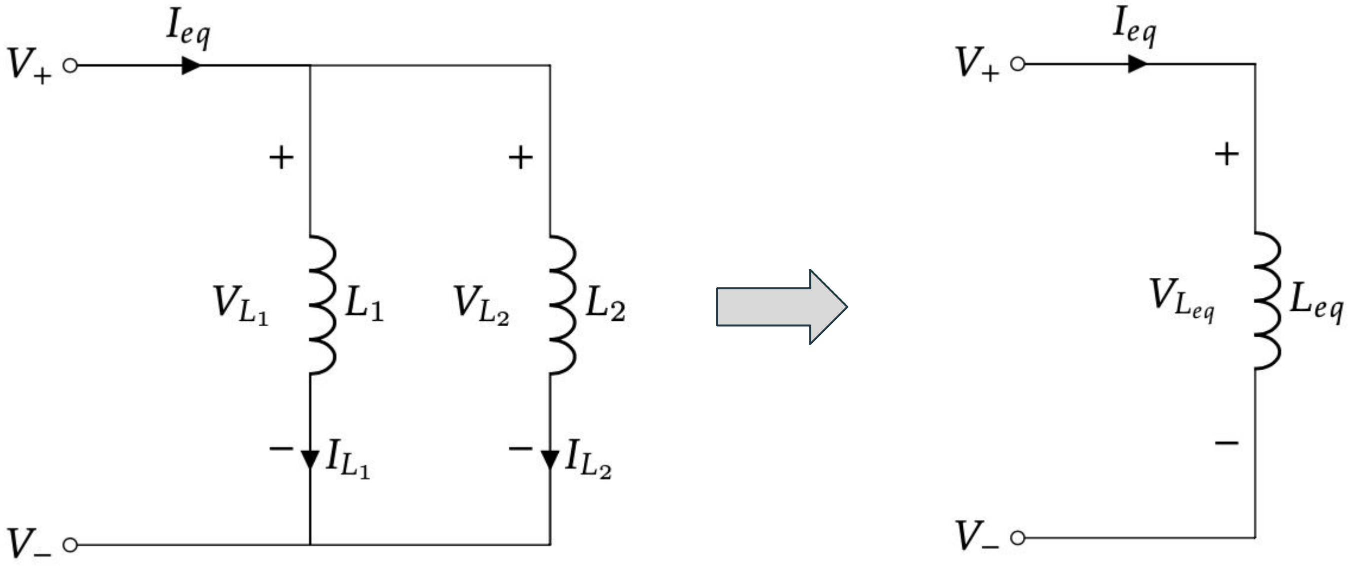


(a) Original circuit

(b) Equivalent circuit for steady state

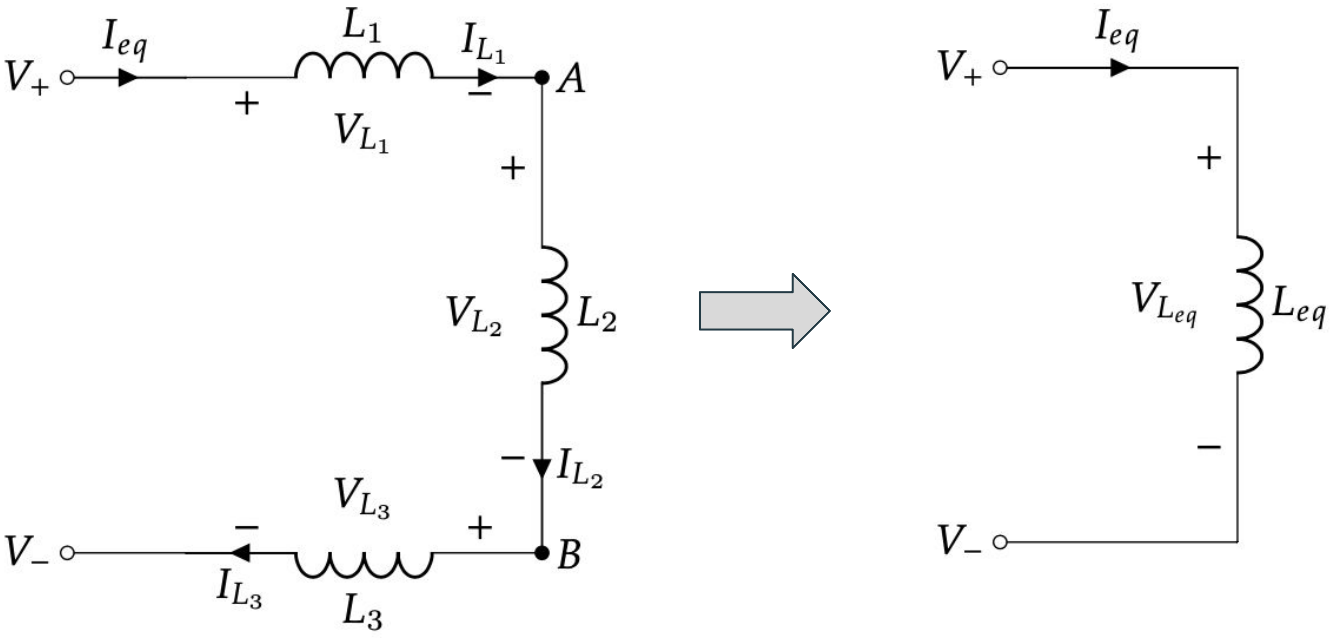
Dis 4B Qa

What is the equivalent inductance for two inductors connected in parallel?



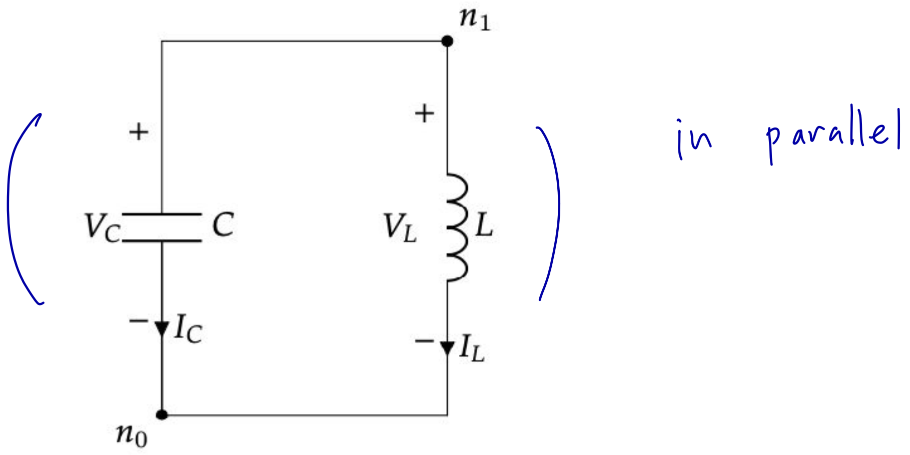
Dis 4B Qa

What is the equivalent inductance for three inductors connected in series?



LC Tank

An LC tank is a circuit with an inductor and capacitor in parallel



As we'll see in Q2, the voltages and currents stay in the tank but will oscillate.

Recap

1. Today we analyzed inductors and their oscillatory behavior when paired with a capacitor.
2. The total energy in the LC Tank circuit was constant and continually oscillated between the inductor and capacitor.
3. The eigenvalues were complex but we saw that complex exponentials correspond to sinusoids.