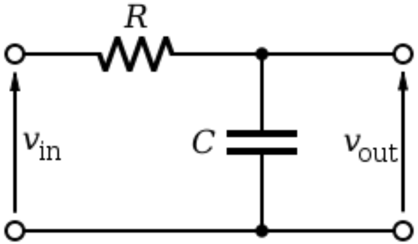
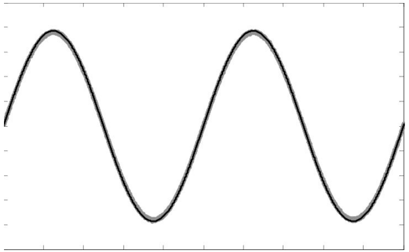
Discussion 5B

Transfer Functions & Filters

Logo credits go to Moses Won





Recap

If the voltages and currents in a linear circuit are **sinusoidal**, then we can use **phasors** to analyze our circuit.

- A linear circuit **does not** change the frequency of a sinusoid.

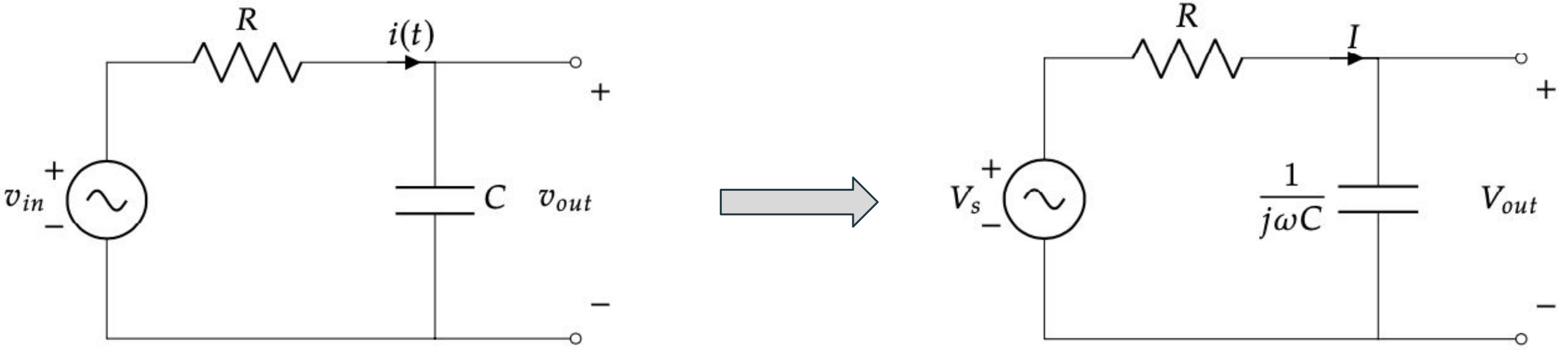
Let's say the voltage $v(t) = A\cos(\omega t + \phi)$, then the phasor $\mathbf{V} = \mathbf{A}\mathbf{e}^{j\phi}$.

We could transform our entire circuit into the phasor domain using impedances!

- Resistors have impedance R
- Capacitors have impedance $1/j\omega C$
- Inductors have impedance $j\omega L$

Quick Example

Given the following circuit in the **time-domain** We can transform it into the **phasor domain**



Then, we can solve for phasor voltage $\mathbf{V}_{\mathbf{out}}$ and convert that back into the time domain.

Ex: If $\mathbf{V}_{\mathbf{out}} = 2e^{j\,\pi/4}$, then $v_{out}(t) = 2\cos(\omega t + \pi/4)$.

Plan for Today

We want to see how our circuits behave for inputs of different frequencies.

Let's define a **transfer function** which will be output to input ratio of the phasors voltages: $H(\omega) = V_{out} / V_{in}$

- The transfer function takes frequencies ω and outputs a complex scalar.
- It's the frequency equivalent of **gain** from 16A: $G = V_{out} / V_{in}$

Ex: $\omega = 20kHz$ is annoying, we can design $H(20kHz) = 0$

Transfer functions can help us design **filters** that block out certain frequencies.

- Many applications such as audio-engineering, WiFi, computer graphics.

Poles and Zeros

$H(\omega) = \frac{j\omega RC}{1 + j\omega RC} \cdot \frac{1 + j\omega R/L}{j\omega L/R} \cdot \frac{1 + j \cdot C}{1 + j \cdot LC}$

Any transfer function can be factored into the following form:

$$H(\omega) = \frac{z(\omega)}{p(\omega)} = \frac{(j\omega)^{N_{z0}}}{(j\omega)^{N_{p0}}} \left(\frac{(j\omega)^n \alpha_n + (j\omega)^{n-1}\alpha_{n-1} + \cdots + j\omega\alpha_1 + \alpha_0}{(j\omega)^m \beta_m + (j\omega)^{m-1}\beta_{m-1} + \cdots + j\omega\beta_1 + \beta_0} \right)$$

$$= K \frac{(j\omega)^{N_{z0}} \left(1 + j\frac{\omega}{\omega_{z1}}\right)\left(1 + j\frac{\omega}{\omega_{z2}}\right) \cdots \left(1 + j\frac{\omega}{\omega_{zn}}\right)}{(j\omega)^{N_{p0}} \left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right) \cdots \left(1 + j\frac{\omega}{\omega_{pm}}\right)}$$

FTA: Any n^{th} degree polynomial has n complex roots

The ω_z are called **zeros** and the ω_p are called **poles.**

Really important for plotting

Where things start to change.

Ex: $$H_2(\omega) = \frac{j\omega\,(1 + j\omega/50)}{(1 + j\omega/100)(1 + j\omega/5)}$$

This transfer function has a zero at 50, and poles at 5, 100, also have a zero at $\omega = 0$,