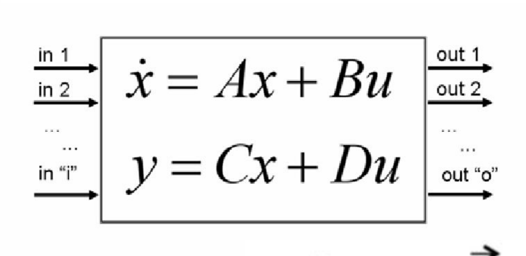
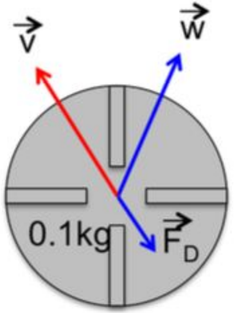
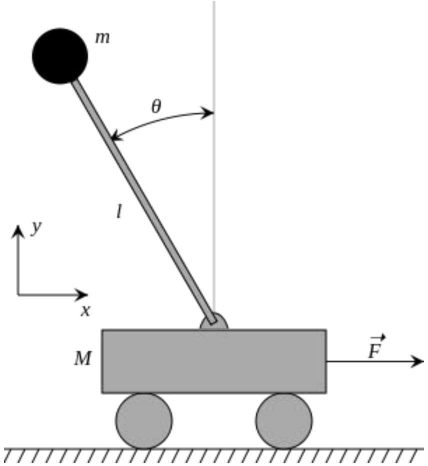
Discussion 6B

State-Space Models

Logo credits go to Moses Won







Discussion Feedback

Any feedback is welcome!

- https://forms.gle/HQtVmncbCj2aj69z9

Recap

We've looked at modeling circuits in both **time and frequency.**

- In the time-domain, we used **differential equations.**
- In frequency, we used **phasors** which were time-invariant.

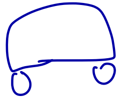
To solve an RC-RC or RLC circuit, we used **multivariate differential equations of the form:**

$$\frac{d}{dt}\vec{x} = A\vec{x} + \vec{b}$$

Today, we'll look at how to create some new models in time that will help us understand other types of systems.

State Space Models

A **state-space model** consists of variables: $x_1, \ldots, x_n$ called **states** and variables $u_1, \ldots, u_m$ called **inputs.** These states are governed by some equations called the **model.**



input : gas pedal

There are two classes of state-space models:

Continuous-Time

$$\frac{d}{dt}\vec{x}(t) = f(\vec{x}(t), \vec{u}(t))$$

$t \in \mathbb{R}^+$

Discrete-Time

$$\vec{x}[t+1] = f(\vec{x}[t], \vec{u}[t])$$

$t = 0, 1, 2, \ldots,$ $t \in \mathbb{N}$

While the two seem similar on paper, we'll learn that they behave quite differently throughout this module.

Linearity

A **state-space model** is **linear** if it can be represented in the form:

Continuous-Time

$$\frac{d}{dt}\vec{x}(t) = \mathbf{A}\vec{x}(t) + \mathbf{B}\vec{u}(t)$$

↑ eigenvalues of A

$x(t) = \alpha_1 e^{\lambda_1 t} + \dots + \alpha_n e^{\lambda_n t}$

Discrete-Time

$$\vec{x}[t+1] = \mathbf{A}\vec{x}[t] + \mathbf{B}\vec{u}[t]$$

Examples:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - 3x_2 + u_1 \\ 4x_1 - 5x_2 - 2u_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$A \quad \vec{x} \quad + \quad B \quad \vec{u}$

multiplied ↙

$$\begin{bmatrix} x_1[t+1] \\ x_2[t+1] \end{bmatrix} = \begin{bmatrix} 3x_1[t]x_2[t] \\ -4x_1[t] + 2u[t] \end{bmatrix}$$