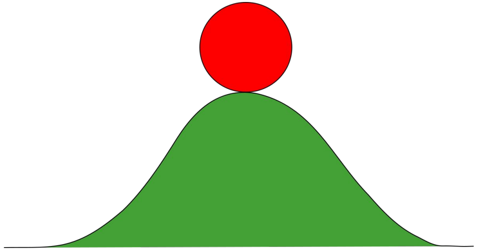
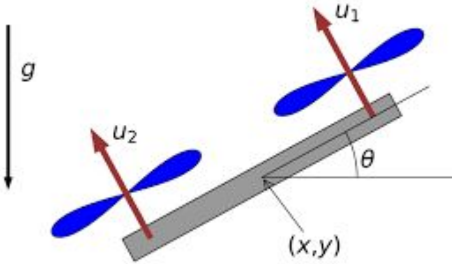
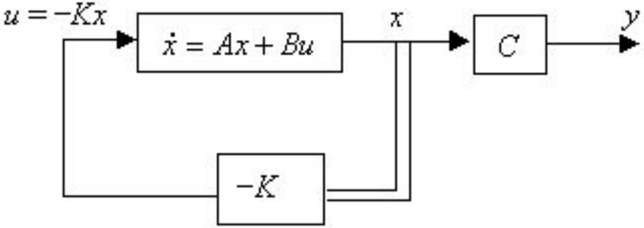
Discussion 8B

Feedback Control

Logo credits go to Moses Won







Recap

Last Wednesday, we analyzed the stability of a state-space model.

- A system is **BIBO Stable** if for every input u, the state **x** is bounded.
- A system is **asymptotically stable** if given zero input, **x** converges to **0.**
- A Continuous-Time System is **stable** if $\Re e[\lambda_i] < 0 \quad \text{for all } i = 1, \ldots .n$
- A Discrete-Time System is **stable** if $|\lambda_i| < 1 \quad \text{for all } i = 1, \ldots .n$

A system is controllable if it can reach any state in its state-space after a finite number of time-steps.

$$C = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

If **C** is full rank, then the system is controllable.

Feedback Control

If a system is unstable, it's possible to stabilize it!

$$\vec{x}[t+1] = A\vec{x}[t] + B\vec{u}[t]$$

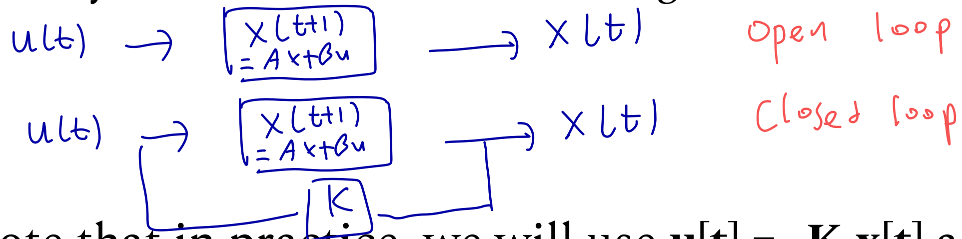
> unstable because $|\lambda_i| \geq 1$ for some i

By inputting **u**[**t**] = **K x**[**t**], the **closed-loop** system will be

$$\vec{x}[t+1] = (A + BK)\vec{x}[t]$$

> now system is stable if the eigs of A+BK, $|\lambda| < 1$

This system is stable when the eigenvalues of **A** + **BK** are stable.



Note that in practice, we will use **u**[**t**] = -**K x**[**t**] as negative feedback.

Controllability

If a system is **controllable**, then it is possible to assign the eigenvalues of **A +BK** anywhere in the complex plane.

- Even if a system is uncontrollable, it may be possible to stabilize it. However, we can no longer place the eigenvalues arbitrarily.
- Proof is out of scope, but if you're interested look up **Controllable Canonical Form**

As we'll see in discussion today, some eigenvalues are "better" than others.